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LETTER TO THE EDITOR

Noise reduction in Eden models: I

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Abstract. We adapt the multiple hit method of noise reduction introduced earlier for diffusion-limited growth to Eden models A and C on the square lattice. The dynamic scaling is improved considerably: we obtain 0.33 ± 0.015 (model A) for the dynamic exponent β of surface roughening with relatively small computing effort. Results for model C support universality.

Among random growth models the simplest one, the Eden model (Eden 1961), continues to attract attention. The growth proceeds on a lattice by occupying the current perimeter sites of a cluster at random (for reviews see Herrmann (1986) and Stauffer (1987)). In spite of its simplicity the model has non-trivial scaling behaviour and has served to develop new concepts also useful in other growth models.

Due to the noise the surface region of an Eden cluster has a finite width w which grows following a power law with an exponent β: $w \sim t^{\beta}$ where t denotes the time of growth (Plischke and Rácz 1984, Family and Vicsek 1985). Kardar *et al* (1986) argued on the basis of a renormalisation group theory of a Langevin equation of surface growth that $\beta = \frac{1}{3}$ in two dimensions. This value was also obtained by Dhar (1986) in an exact solution of a modified Eden model. Numerical evidence has also been given for $\beta = \frac{1}{3}$ (Jullien and Botet 1985, Plischke and Rácz 1985, Hirsch and Wolf 1986, Zabolitzky and Stauffer 1986). However, due to very slow convergence, it takes quite a lot of computing effort to obtain accurate values for β.

Recently, for diffusion-limited growth, a numerical method was developed to control the fluctuations. A perimeter is getting part of the cluster only if it has been visited m times (Tang 1985, Szép *et al* 1985). The increasing hitting number m results in a reduction of noise. It has been argued that for diffusion-limited aggregation the noise reduction leads to a faster convergence to the asymptotic shape of the clusters (Kertész and Vicsek 1986). In this letter we apply the noise reduction method to the Eden model.

We implemented the following algorithm: a counter is put on each perimeter site of the cluster. If a given perimeter site is chosen randomly, the corresponding counter is set $i+1$ if it was at value i before. In the case where the counter reaches the value m the site is added to the cluster; the counters at the possibly new-born perimeter sites are set to be zero while the other counters remain in their current state. This procedure is applied to Eden model A in the terminology of Jullien and Botet (1985). We also carried out calculations for model C where first an occupied site on the boundary of the cluster is picked randomly and then one of its empty neighbour sites is filled. In this case we put the counters on the boundary sites.

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In order to get rid of the effect of the anisotropic shape of the clusters (see Wolf 1987) we let them grow on flat substrates of length L (Jullien and Botet 1985). Helical boundary conditions are used at the boundaries perpendicular to the substrate. The total number N of particles in the cluster is a measure of the time t of growth: $t = N/L$. At integer and half-integer times we monitor the number of perimeter sites and the width w of the surface region defined as

$$w = (\langle r^2 \rangle - \langle r \rangle^2)^{1/2} \tag{1}$$

where r is the distance of a perimeter site from the substrate. The angular brackets denote averaging over all perimeter sites. In this letter we restrict ourselves to substrates parallel to an axis of the square lattice. We carried out Monte Carlo simulations of models A and C for $L = 120$, 240, 480, 960 with $m = 1$, 2, 4, 8, 16, 32, 64. Averages over n runs were taken where n varied from 1000 ($L = 120$) to 35 ($L = 960$).

It is instructive to follow how an arbitrary surface configuration evolves when noise reduction with $m = \infty$ is switched on suddenly. Figure 1 demonstrates that within a very short time all steps higher than one lattice constant, all holes and overhangs are removed from the surface configuration. This smoothening effect of noise reduction can already be observed for small m, where holes, overhangs and steps higher than one lattice constant are not removed entirely but are increasingly suppressed the larger m is. In this sense noise reduction plays the role of an effective surface tension†.

If there are no overhangs, holes and steps higher than one lattice constant the number of perimeter sites is equal to the number of adsorption sites of the substrate even if the surface is not parallel to the substrate but fluctuates. Correspondingly, we observe that the number of perimeter or boundary sites N_p approaches L if we increase m. For $m = 4$ the difference $N_p - L$ is already less than a quarter of that for $m = 1$.

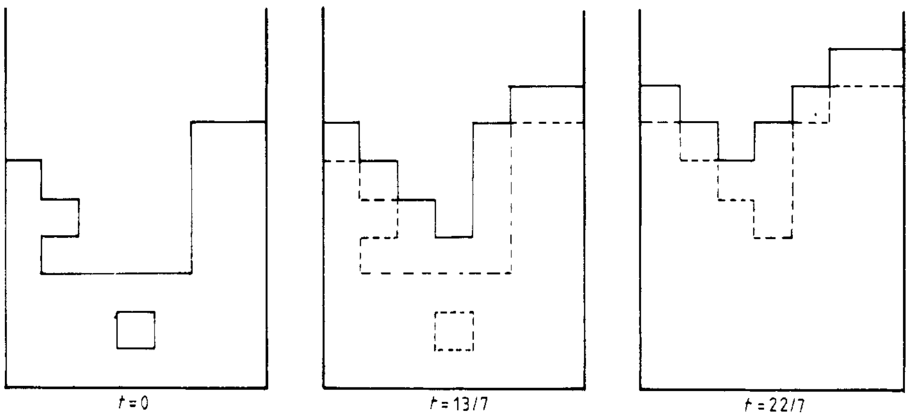

Figure 1. Mechanism of noise reduction. At $t = 0$ noise reduction with $m = \infty$ is switched on. The holes, overhangs and steps higher than one lattice unit disappear and the number of perimeters is L. If $m < \infty$ the intrinsic width is still small and therefore the corrections to scaling are suppressed.

† This can be the explanation of Nittmann and Stanley's (1986) results on diffusion-limited growth on lattices. Due to their special boundary condition (R C Ball private communication) they got patterns *with* noise reduction but *without* surface tension which were similar to those obtained in experiments *with* surface tension. However, with slightly different boundary conditions the instability to anisotropy, which is very characteristic for that system, governs the process (Kertész and Vicsek 1986).

Why do we expect that noise reduction improves the dynamic scaling behaviour? According to Hirsch and Wolf (1986) the width seems to be influenced by the relaxation of the perimeter. More precisely, there are two contributions to the width (Zabolitzky and Stauffer 1986). One is the intrinsic width due to overhangs, holes and high steps and can be measured by the number of perimeter sites. In addition, long wavelength fluctuations of the surface position similar to capillary waves (Jasnow 1985) emerge. It is the dynamical behaviour of this second contribution which is responsible for dynamic scaling while some corrections to scaling stem from the development of the intrinsic width. If noise reduction is switched on, the amplitude of the intrinsic width is suppressed and therefore the dynamic scaling should show up already for small system sizes.

We observed in fact a definite improvement in the scaling behaviour (figure 2). Already for $L=120$, a small length where scaling is not yet valid for $m=1$, we have nice 'straight' regions on the $\log(w)$ against $\log(t)$ plots if m is sufficiently large ($m>4$). The curves for different m values have fairly parallel regions and indicate that noise reduction does not change the universality class of Eden growth. This conclusion is in particular supported by our data for $L=960$ (Kertész and Wolf 1987).

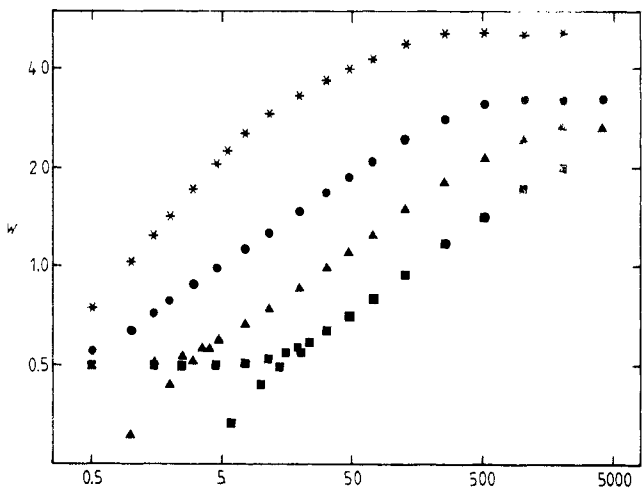

Figure 2. Log-log plot of width against time (model A). $L=120$, $m=1$ (*), 4(●), 16(▲), 64 (■). For $m=16$ and 64 the 'half-filled' (upper curves) and the 'fully filled' (lower curves) layers can be distinguished.

We looked at the consecutive slopes (effective values of β) for different m values (figure 3). For the νth β value we fitted a straight line to four equidistant points from figure 2 with half-integer times: $t_i = 2^{\nu+i} - \frac{1}{2}$, $i=1,\ldots,4$. For $m>4$ the $\beta(t)$ curves have a maximum which is shifted rightwards corresponding to the fact that stronger noise reduction results in slower convergence to the equilibrium width.

In figure 4 $\beta(t)$ is plotted for fixed $m=64$ and four different L values. While for small times the curves are independent of L, the maximum becomes broader as L increases. In the scaling limit L going to ∞ the function $\beta(t)$ should approach a plateau representing the exponent β. We tried to extrapolate to this limit by plotting the maxima $\beta_{\max}$ of the $\beta(t)$ curves against $1/L$ (insert in figure 4). A best linear fit gives

$$\beta = 0.33 \pm 0.015.$$

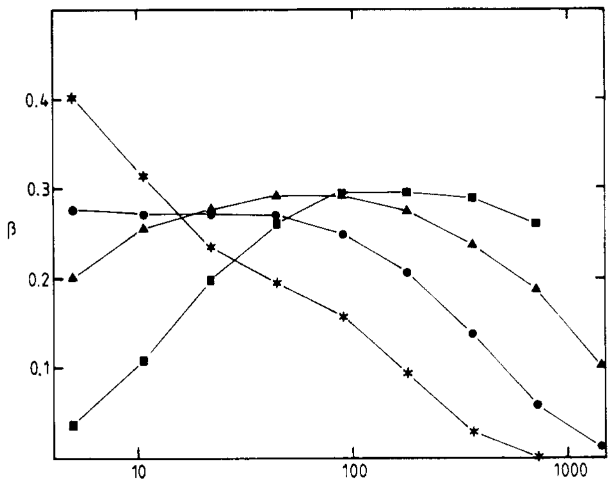

Figure 3. Consecutive slopes for β. $L = 120$, $m = 1$ (∗), 4 (●), 16 (▲), 64 (■).

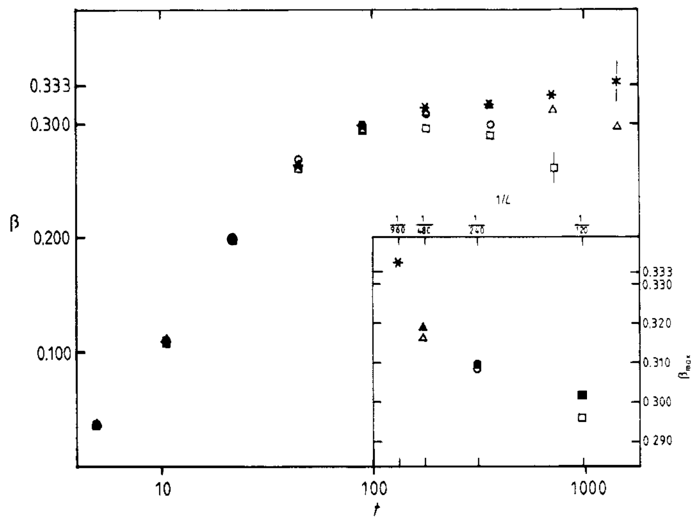

Figure 4. Consecutive slopes for model A with $m = 64$ as a function of $L = 120$ (□), 240 (○), 496 (△), 960 (∗). The insert shows the β_{max} against $1/L$ plot. Open symbols and ∗, model A; full symbols, model C.

Similar results had been obtained previously for model A without noise reduction (Hirsch and Wolf 1986, Zabolitzky and Stauffer 1986). However, we have got the same numerical accuracy as Zabolitzky and Stauffer (1986) with orders of magnitude less memory (they used a Cray 2) and also the computing time was reduced by a factor of two. We needed for the calculations presented here about 20 h CPU time on the Cyber 76.

We carried out the same calculations for model C which was shown in a study of Jullien and Botet (1985) to have better dynamic scaling properties than model A. This is not so if noise reduction is applied. The β_{max} values for $m = 64$ are plotted in the insert of figure 4. Since in the algorithm of model C single bit handling is less feasible and costs more computer time than in model A, we did not study $L = 960$ here. Although Meakin *et al* (1986) found $\beta = 0.307 \pm 0.007$, which is somewhat in conflict with the analytical prediction $\beta = \frac{1}{3}$, our result supports universality and shows a systematic trend towards larger β values for increasing L. An extrapolation of β_{max} to the infinite system size leads to $\beta = 0.326 \pm 0.015$.

As a generalisation of the above ideas one can speculate that in the dynamic scaling of interface roughening, e.g. in the Ising model (Bausch *et al* 1981), important corrections come from the intrinsic width. In this case lowering the temperature would correspond to noise reduction. Finally we mention that noise reduction raises a number of interesting questions: can the hitting number m be considered as a scaling variable, how is the anisotropy influenced, is noise reduction related to renormalisation group ideas, etc. We shall deal with some of these questions in a forthcoming publication (Kertész and Wolf 1987) and we also plan to apply this powerful method to the three-dimensional case.

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References

Bausch R, Dohm V, Janssen H K and Zia R K P 1981 *Phys. Rev. Lett.* **47** 1937
Dhar D 1986 *STATPHYS 16, Boston, 10-16 August* Poster
Eden M 1961 *Proc. 4th Berkeley Symp. on Mathematical Statistics and Probability* vol 4, ed F Neyman (Berkeley, CA: University of California Press) p 223
Family F and Vicsek T 1985 *J. Phys. A: Math. Gen.* **18** L75
Herrmann H J 1986 *Phys. Rep.* **136** 153
Hirsch R and Wolf D E 1986 *J. Phys. A: Math. Gen.* **19** L251
Jasnow D 1985 *Rep. Prog. Phys.* **47** 1059
Jullien R and Botet R 1985 *J. Phys. A: Math. Gen.* **18** 2279
Kardar M, Parisi G and Zhang Y C 1986 *Phys. Rev. Lett.* **56** 889
Kertész J and Vicsek T 1986 *J. Phys. A: Math. Gen.* **19** L257
Kertész J and Wolf D E 1987 in preparation
Meakin P, Jullien R and Botet R 1986 *Europhys. Lett.* **1** 609
Nittmann J and Stanley H E 1986 *Nature* **321** 663
Plischke M and Rácz Z 1984 *Phys. Rev. Lett.* **53** 415
—— 1985 *Phys. Rev.* A **32** 3825
Stauffer D 1987 *Proc. Workshop on Percolation Theory and Ergodic Theory of Infinite Particle Systems, Minnesota, February, 1986* ed H Kesten Preprint
Szép J, Cserti J and Kertész J 1985 *J. Phys. A: Math. Gen.* **18** L413
Tang C 1985 *Phys. Rev.* A **31** 1977
Wolf D E 1987 *J. Phys. A: Math. Gen.* **20** 1251
Zabolitzky J G and Stauffer D 1986 *Phys. Rev.* A **34** 1523